



Erratum

Erratum to “A general algorithm for exact simulation of multicomponent aggregation processes” [J. Comput. Phys. 177 (2002) 418–449]

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Lushnikov's population balance equation describing the aggregation of particles composed of at most two-components is [1]

$$\begin{aligned} \frac{d\hat{c}(u, v; t)}{dt} = & \frac{1}{2} \int_0^u \int_0^v K(u', v' | u - u', v - v') \hat{c}(u', v'; t) \hat{c}(u - u', v - v'; t) dv' du' - \hat{c}(u, v; t) \int_0^\infty \\ & \times \int_0^\infty K(u, v | u', v') \hat{c}(u', v'; t) du' dv'. \end{aligned} \quad (1)$$

In the published paper [2], a solution for the special case where $K(u, v | u', v') = \beta = \text{const.}$ was presented, subject to the initial distribution corresponding to a mixture of two populations of homogeneous particles, each exponentially distributed in size

$$\hat{c}(u, v; 0) = c_1 \lambda_1 e^{-\lambda_1 u} \lambda_2 \delta(\lambda_2 v) + c_2 \lambda_2 e^{-\lambda_2 v} \lambda_1 \delta(\lambda_1 u). \quad (2)$$

Although the cumulative distribution $G(u, v; t) = \int_0^u \int_0^v \hat{c}(u, v; t) dv du$ presented as Eq. (54) is correct as written, the solution for the concentration density function presented as Eq. (A.6) has a typographical error. The correct expression is

$$\begin{aligned} \hat{c}(u, v; T) = & \frac{4c_0 \lambda_1 \lambda_2}{(2 + T)^2} \left\{ x_1 \delta(\lambda_2 v) e^{-\lambda_1 v_1 u} + x_2 \delta(\lambda_1 u) e^{-\lambda_2 v_2 v} + 2x_1 x_2 \Theta I_0 \left(2\Theta \sqrt{x_1 x_2 \lambda_2 \lambda_1 uv} \right) \right. \\ & \left. + x_1 x_2 \Theta e^{-\lambda_1 v_1 u - \lambda_2 v_2 v} \left[\left(\sqrt{\frac{x_2 \lambda_2 v}{x_1 \lambda_1 u}} + \sqrt{\frac{x_1 \lambda_1 u}{x_2 \lambda_2 v}} \right) \times I_1 \left(2\Theta \sqrt{(x_1 \lambda_1 u)(x_2 \lambda_2 v)} \right) \right] \right\}, \end{aligned} \quad (3)$$

where

DOI of original article: [10.1006/jcph.2002.7017](https://doi.org/10.1006/jcph.2002.7017).

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$$\begin{aligned}\Theta &= \frac{T}{2+T}, \\ v_i &= 1 - \Theta x_i, \quad i = 1, 2, \\ x_i &= \frac{c_i}{c_0}, \quad i = 1, 2, \\ c_0 &= c_1 + c_2, \\ T &= c_0 \beta t\end{aligned}$$

and $I_n(x)$ is the modified Bessel function [3]. In the published paper, the factors of Θ multiplying the modified Bessel functions were absent. Note that Eq. (3) reverts to Eq. (2) as $T \rightarrow 0$, as one would expect.

Finally, there is a typo on p. 432, Eq. (45) should read

$$c(m, n; t) = \binom{m+n}{n} \left(\frac{c_1}{c_0}\right)^m \left(\frac{c_2}{c_0}\right)^n c(m+n, t), \quad c_0 = c_1 + c_2. \quad (4)$$

All results presented in the published paper employ the (correct) expressions presented here.

References

- [1] A.A. Lushnikov, Evolution of coagulating systems. III. Coagulating mixtures, *J. Colloid Interface Sci.* 54 (1976) 94.
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- [3] M. Abramovitz, I.A. Stegun, *Handbook of Mathematical Functions*, Dover, New York, NY, 1965.